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Interoperable Global and Local Indexing of Discrete Global Grid Systems Based on the ISEA Projection for Efficient Storage, Processing and Transmission

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Abstract:

The Icosahedral Snyder Equal-Area (ISEA) projection (Snyder, 1992) has been used for constructing several Discrete Global Grids Hierarchies (DGGH) such as ISEA3H (aperture 3 hexagonal) (Sahr et al., 2003). However, the implementation of these grids and interoperability between them has been limited by several factors. Developers faced difficulties understanding the 1992 paper sufficiently well to implement it in software. Technical errors in subsequently published papers introduced further confusion. The Earth model has been mapped to the ISEA projection in different ways, often not correctly establishing the correspondence between geodetic latitude and latitude on the authalic sphere for which the projection is defined. Different grids may also use a different orientation of the icosahedron for the projection. We highlight that grids with more regular zones compared to ISEA result from mapping great circles from the icosahedron vertices (van Leeuwen and Strebe, 2006) rather than from the face centroids – we refer to this projection as "IVEA", which might obsolete ISEA.

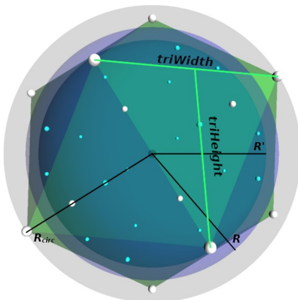

Figure 1. The authalic (R) and inner (R') spheres of the ISEA projection, shown with the icosahedron and its circumsphere (R_{circ})

We present clarifications on key concepts of the ISEA projection, such as the relationships between the icosahedron and the authalic sphere, circumsphere and inner sphere. We clarify details specific to the icosahedron, since the original 1992 publication defined a wider class of polyhedral globe projections. We explain the importance of mapping geodetic latitude to authalic latitude for ISEA/IVEA grids to be truly equal-area. We propose an interoperable orientation for ISEA/IVEA grids offsetting the projection 0.05° West relative to the common orientation symmetrical about the equator (Sahr et al., 2003), allowing to maintain a single vertex on land when the authalic latitude conversion is performed. We also mention contributions by the

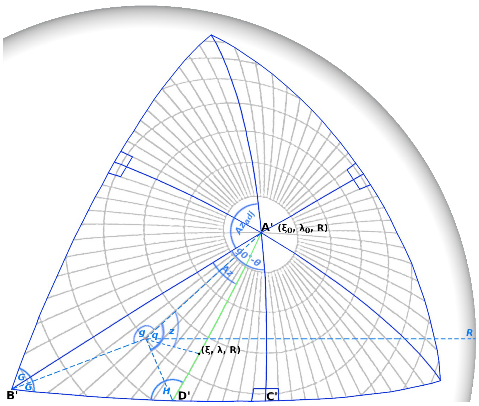

Figure 2. The six spherical triangles of the ISEA projection on the face of the authalic sphere corresponding to right triangles on a face of the icosahedron on which they are projected.

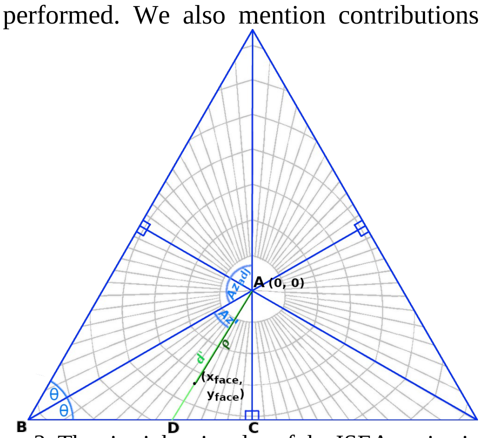

Figure 3. The six right triangles of the ISEA projection on the face of the icosahedron corresponding to spherical triangles on the authalic sphere.

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contributing author to the PROJ software library[1] enabling support for the inverse projection, initially based on open-source code from geogrid[2], making the ISEA projection usable in popular software such as GDAL and QGIS. We also note an open-source implementation of ISEA3H / 9R and IVEA3H / 9R in DGGAL[3] from the same author, including a closed form of the inverse projection, as well as a linear algebra approach[4] avoiding costly trigonometric functions and thus improving performance.

We define a Coordinate Reference System (CRS) derived from translating, rotating, shearing, and scaling the ISEA planar projection to a 5 x 6 Cartesian space where a 1x1 space corresponds to each of the ten rhombuses formed by two icosahedron triangles joined at their base, facilitating a number of operations such as indexing.

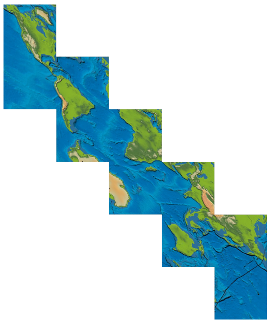

Figure 4. GEBCO 2014 Grid shown in 5x6 Cartesian rhombic space of ISEA projection

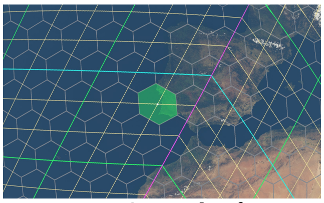

Figure 5. Area value for a ISEA3H level 6 hexagon corresponding to an ISEA9R level 3 node value shown over NASA Earth Observatory's Blue Marble Next Generation

Multiple approaches to indexing hexagonal grids have been proposed in the past to address the additional challenges they pose. Most of these have disadvantages such as long textual identifiers not ideal for transmission of large lists of zones, difficulty to map identifiers to compact integer types for internal processing, as well as complexity increasing with grid depth for resolving an identifier to a particular zone and associated properties. The relatively simple approach we propose leverages the dual relationship between ISEA3H / IVEA3H and ISEA9R / IVEA9R (aperture 9 rhombic) DGGHs for defining a global indexing scheme. This results in compact textual identifiers with a corresponding 64-bit integer up to a very detailed refinement level.

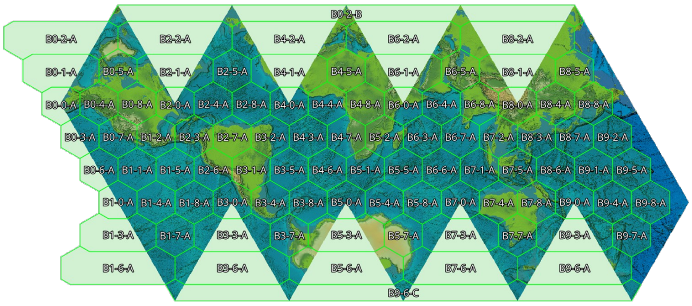

Figure 6. Level 2 zone identifiers of ISEA3H DGGRS overlaid on visualization of GEBCO 2014 Grid

In conjunction with this global indexing, we define a deterministic ordering of sub-zones (zones of a finer refinement level at least partially contained within a coarser parent zone) based on scanlines for both ISEA3H / IVEA3H and ISEA9R / IVEA9R, which allows for internal addressing within a single zone. This sub-zone ordering enables efficient storage, processing and transmission of both vector and raster data quantized and referenced to ISEA grids using the formats and API defined by the candidate OGC API – Discrete Global Grid Systems Standard[5]. The clear definition of an interoperable Discrete Global Grid Reference System (DGGRS) with truly equal-area hexagonal zones, combined with the ability to efficiently store and transmit both vector and raster data quantized to sub-zones, paves the way for DGGS to revolutionize how geospatial data is shared, integrated, visualized and analyzed, with major gains in terms of scalability, simplicity and performance.

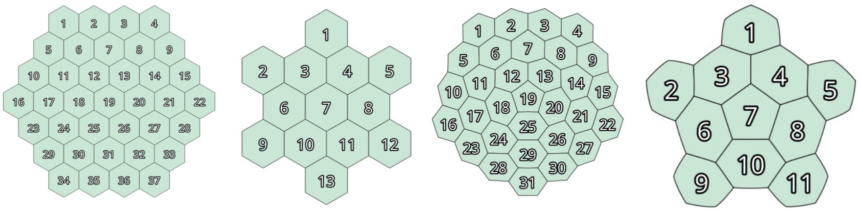

Figure 7. Ordering of sub-zones for hexagonal (left) and pentagonal (right) parent zones at relative depths of 3 (first and third from left) and 2 (second and last)

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van Leeuwen, D., and Strebe, D., 2006: A “Slice-and-Dice” Approach to Area Equivalence in Polyhedral Map Projections. Cartography and Geographic Information Science, 33(4), 269–286. https://doi.org/10.1559/152304006779500687

1 https://proj.org/en/latest/operations/projections/isea.html

2 https://github.com/mocnik-science/geogrid/

3 https://dggal.org

4 https://brsr.github.io/2021/08/31/snyder-equal-area.html

5 https://docs.ogc.org/DRAFTS/21-038r1.html

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