International Cartographic Association
Association Cartographique Internationale

Towards a discrete generalization of cartographic projections

Luke Bergmann [a,*], David O'Sullivan [b,c]

[a] *University of British Columbia, Canada – luke.bergmann@gmail.com*
[b] *University of Auckland, New Zealand – david.osullivan@auckland.ac.nz*
[c] *Geospatial Stuff, New Zealand – dosull.github.io*

* Corresponding author

Keywords: discrete projections, discrete geometry, more relations among more spaces

Abstract: Cartographic software supports a large number of cartographic projections, families of parameterized transformations many of which are used to map coordinates between the sphere-like Earth and the flat two-dimensional media of most maps. Many ingenious transformations suitable to varied analytical and regional contexts have been devised and then made available for general use through software libraries. Mathematically and computationally, a wide range of methods are employed in the process.

Yet we suggest there are many sorts of transformations and spaces that are not widely accessible to cartographers in the present. For example, Tobler (1963) long argued that cartograms are themselves projections. But cartograms are today largely made through implementations of algorithms with little practical connection to projection libraries. While the cartogram code and inputs may be shared such that others may potentially project new datasets into the cartogram, this is not the same as having the transformation defined by the cartogram be represented itself as a projection and communicated as such among cartographers and their software packages. A second type of projection historically largely inaccessible to computer-facilitated cartography includes those where the scale is not expressed in units of map distance per linear surface distance, but map distance per time or cost or other unit (Tobler 1961; Gattrell 1983; Muller 1984; L'Hostis and Abdou 2021). These transformations are sometimes effectively 'distance cartograms' and thus can be related to the [areal] cartogram example above, but some of these maps may solely preserve radial distance from a single point (see Payne and McGlynn 2024) and may be of non-metric spaces. Third, we have maps and transformations of spaces that may exist in some tension with the abstract absolute spaces assumed by much of western cartography. These can include maps of literary spaces and dream spaces; art maps; and maps that may challenge or even selectively participate in hegemonic cartographic traditions but make use of software to engage visually (Caquard and Fiset 2014; Iosifescu Enescu 2015; Rose-Redwood et al 2020; Bergmann and Lally 2021). Finally, even for mainstream cartographic projections, ensuring that projections (and projected data) are able to be interpreted and handled in a consistent manner by two individuals using different software has been a nontrivial task requiring the ongoing development of both software and projection description languages (e.g., proj strings and WKT-CRS).

We propose that practical cartographic outputs, cartographic communications among practitioners, cartographic theory, and perhaps even cartographic imaginations themselves might benefit from 'generalized projection' approaches that would place more of the aforementioned cartographic endeavours on equal footing. We suggest one such family of approaches may be found by building upon an observation by Tobler (1977)—and by many others in related contexts, including, at least implicitly, by all those who have georeferenced an aerial image—that cartographic projections can be approximated numerically through discrete (and thus subsequently interpolated) mappings between spaces. Over intervening decades, not only has the cost of computation plummeted, but the language and machinery of discrete/computational geometry has developed considerably (Crane 2017). There are opportunities to apply the latter and extend them to better handle the range of cartographic cases as sketched above as different examples of a generalized, discrete formulation of projections.

We began with efforts in Bergmann and O'Sullivan (2017, 2019) to sketch a conceptual framework and notation that we might be able to build upon to represent a class of generalized discrete cartographic transformations. Some such transformations can be relatively straightforward (e.g., one-to-one smooth mappings back and forth between latitude-longitude spherical coordinates and a single region of the x-y plane). In such transformations, we record how the vertices of two triangular meshes (in two different spaces) can be mapped from one to the other, with linear interpolations used to determine how arbitrary points in one space might transform to the other. But a more general formulation adequate to the situations described above must also allow for greater complexity, supporting, for example, many-to-many transformations as well as disconnected and discontinuous spaces.

We have since modified our approach to this research on the supposition that the challenges of generalized discrete projections may be profitably articulated and met though a more holistic approach, one in which various pieces of the

overall puzzle are worked out iteratively in conversation with each other, not linear progression. As such, here, we present progress and invite audience feedback on aspects of several interrelated issues, such as:

- Conceptualizing these transformations and expressing them through useful symbolic notation;
- Identifying and adapting algorithms and data structures appropriate to these transformations and working on implementations. These include not only for the basic transformations of data using discrete projections but also to handle emerging requirements to analyse and process discrete transformations to meet various expectations of cartographers. An example is seen in mappings of travel times, where spaces may be inverted or folded; in such situations, the cartographer needs tools to be able to choose which of the overlapping spaces to display and how;
- Characterizing the advantages and disadvantages of discrete projections, in general as well as in the cases of particular projection implementations. Disadvantages include uncertainty, added distortion, and trade-offs between accuracy and volume of storage;
- Representing individual transformations in data formats appropriate for storage, communication, and at least in restricted cases, use through current software frameworks (such as QGIS and GDAL via proj's TINshift and/or hgridshift transformations); and,
- Identifying examples of types of maps and areas of study in which such a generalized discrete approach to transformations are (or might be) useful. Such examples have provided (and likely will again provide) impetus to further develop aspects of discrete projections.

Our style in this presentation strives for the intuitive and illustrative, as we seek to briefly sketch an evolving whole of research into discrete projections as we currently imagine them. We hope to encourage others to comment, question, and perhaps work on interconnected or parallel problems.

Acknowledgements

LB would like to thank the Canada Research Chairs and the Canada Foundation for Innovation for support of this research programme.

References

Bergmann, L., O'Sullivan, D. 2017. Computing with many spaces: Generalizing projections for the digital geohumanities and GIScience. *Proc. GeoHumanities'17: 1st ACM SIGSPATIAL Workshop*. dl.acm.org/doi/10.1145/3149858.3149866

Bergmann, L., O'Sullivan, D. 2019. Enabling spaces by GeoComputation. *Proceedings of GeoComputation 2020*. Queenstown, New Zealand, 18-21 September 2019. https://doi.org/10.17608/k6.auckland.9848699.v2

Bergmann, L., Lally, N. 2021. For geographical imagination systems. *Annals of the American Association of Geographers*, Vol. 111, Issue 1, pp. 26–35.

Caquard, S., Fiset, J.P. 2014. How can we map stories? A cybercartographic application for narrative cartography. *Journal of Maps*, Vol. 10, Issue 1, pp. 18–25.

Crane, K. 2018. A Glimpse into Discrete Differential Geometry. *Notices of the AMS*. pp.1153-1159.

Gatrell, A.G. 1983. *Distance and Space: A Geographical Perspective*. Oxford University Press.

Iosifescu Enescu, C.M., Montangero J., Hurni, L. 2015. Toward dream cartography: Mapping dream space and content. *Cartographica*. Vol. 50, Issue 4, pp. 224–37.

L'Hostis, A., Abdou, F. 2021. What is the shape of Geographical Time-Space? A three-dimensional model made of curves and cones. *ISPRS International Journal of Geo-Information*, Vol. 10, Issue 5, pp. 340.

Muller, J.-C. 1984. Canada's Elastic Space: A Portrayal of Route and Cost Distances. *The Canadian Geographer / Le Géographe Canadien*. Vol. 28, Issue 1, pp. 46-62.

Payne, W.B., McGlynn, E. 2024. Relational Reprojection Platform: Non-linear distance transformations of spatial data in R. *Environment and Planning B: Urban Analytics and City Science*, Vol. 51, Issue 2, pp. 546-552.

Rose-Redwood, R, Barnd, N.B., Lucchesi, A.H., Dias, S., Patrick, W. Decolonizing the Map: Recentering Indigenous Mappings. *Cartographica*, Vol. 55, Issue 3, pp.151-162

Tobler, W.R. 1961. *Map Transformations of Geographic Space*. PhD Dissertation. University of Washington.

Tobler, W.R. 1963. Geographic Area and Map Projections. *Geographical Review*, Vol. 53, No. 1, pp. 59–78.

Tobler, W.R. 1977. Numerical approaches to map projections. *Beitrage zur theoretischen Kartographie Festschrift für Erik Amberger*. Vienna. pp. 51-64.