International Cartographic Association
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Approximation of the best conformal map projection for the territory of Hungary

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Keywords: Chebyshev projection, oblated stereographic, conformal mappings

Abstract:

The national coordinate system in Hungary is suffering from the distortion of its triangulation network. Instead of transforming our more accurate GPS measurements into an old, distorted grid, an idea emerged to modernize our network. Because the ellipsoid of the present state plane coordinate system is not WGS84, this would necessitate to change our map projection.

Therefore, an exploratory research was conducted to find a suitable map projection for Hungary. Besides the obvious requirement of conformality, we sought a map projection, in which linear scale l does not exceed $1 \pm 1/10\,000$. Until now, only one (now obsolete) multi-zonal system satisfied this criterion for Hungary.

Among usual map projections, we could not find any candidate that could satisfy our requirements using only a single zone. However, Juhász (2008) found that the best possible 'traditional' map projection for Hungary is the Hotine projection, for which $0.999\,888\,83 < l < 1.000\,111\,18$. This is very close to fulfil our expectations, but still fail short.

These were the motivations to examine the possibility to develop a 'tailor-made' map projection for Hungary in hope of a single-zone coordinate system with acceptable distortions.

Hungarian surveying practice traditionally uses the model of 'double projection', a technology shared only with a few European countries. This means that an auxiliary sphere is used between the ellipsoid and the plane. The Gaussian conformal sphere (Gauß, 1843) has practically no distortion with respect to the ellipsoid, so it enables us to derive our map projection for the sphere with significantly simpler equations.

It is well-known from the analysis of complex numbers that every differentiable (holomorphic) function of the complex plane is conformal and vice versa. Furthermore, all differentiable complex functions can be approximated to any desired accuracy by complex polynomials within some convergence radius. This means that the best conformal map projection can be approximated by a complex polynomial from any arbitrary conformal projection. My results implied that that the choice of the original map projection to be modified by the complex polynomial does not significantly influence the final result. The oblique stereographic projection was chosen, because it is already (albeit marginally) used by Hungarian surveyors, so it should be familiar, and the oblated stereographic projection (i.e., the composition of the stereographic projection and a complex polynomial) is implemented in major projection engines like PROJ. See the process in Fig. 1.

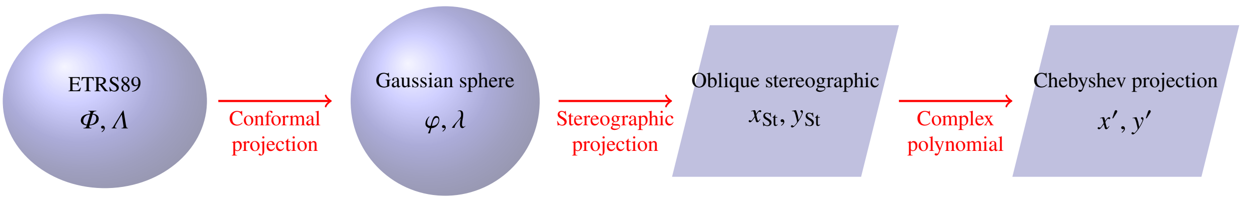

Figure 1. Method of the projection

The complex polynomial of the last step can be described this way:

$$z = y_{\mathrm{St}} + \mathrm{i}x_{\mathrm{St}} \tag{1}$$

$$z' = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + \dots \tag{2}$$

To calculate the optimal coefficients a_i, we need to compute the linear scale of the new map projection (l_{G} and l_{St} are the linear distortions of the Gaussian sphere and the stereographic projection):

$$l = l_{\mathrm{G}} l_{\mathrm{St}} \left| a_1 + 2a_2 z + 3a_3 z^2 + 4a_4 z^3 + \dots \right| \tag{3}$$

Abstracts of the International Cartographic Association, 12, 57, 2026
European Cartographic Conference – EuroCarto 2026, 8–11 September 2026, Brno, Czech Republic.
https://doi.org/10.5194/ica-abs-12-57-2026

The theorem of Chebyshev says that the maximum and minimum linear scale has the least possible difference, if l is constant along the boundary of the region (Gravé, 1911). Let this constant be unit. Therefore (along the border line):

$$\ln l = \ln\left(l_{\mathrm{G}} l_{\mathrm{St}}\right) + \ln\left|a_1 + 2a_2 z + \ldots\right| = 0 \tag{4}$$

Locate equally distributed points (ca. 50 m) along the border of Hungary, and solve coefficients a_i from the non-linear least squares problem:

$$\sum \ln^2 l = \sum \left[\ln\left|a_1 + 2a_2 z + \ldots\right| - \ln\left(1/l_{\mathrm{G}} l_{\mathrm{St}}\right)\right]^2 \to \min \tag{5}$$

We should note that $\ln|w|$ is not differentiable, but writing it as the real function $1/2 \ln(\Re^2 w + \Im^2 w)$, it becomes differentiable in terms of the real and imaginary parts of the coefficients. Double floating-point representation enables us to compute the coefficients up to a_{35}. l varied between 0.999 929 16 and 1.000 070 84 inside Hungary. On the other hand, this map projection turned out to be useless, because it had singularities close to the Hungarian border. The reason is that the border of Hungary is a polygon, i.e., not differentiable. Lower-degree polynomials had their singularities farther from the territory of the country.

It was decided that 10% loss of optimality is acceptable. This could be achieved already by a seventh-degree polynomial. However, the coefficients a_i resulting from the previous calculations are no longer optimal, as theorem of Chebyshev used for the derivation is only valid for the best map projection. Nevertheless, they are close to optimality, so they can be used as starting values for a direct minimum search.

Minimizing $\max l / \min l$, i.e., $\ln \max l - \ln \min l$ directly was hard, as the cost function is not differentiable (thus, the inefficient downhill simplex method was the only choice) and had incredibly many local minima. Even by starting close from the global minimum, plenty of repeated runs were necessary to find a sufficiently stable result.

The recommended map projection had $0.999\,922\,38 < l < 1.000\,077\,62$. Its distribution of linear scale can be seen in Fig. 2.

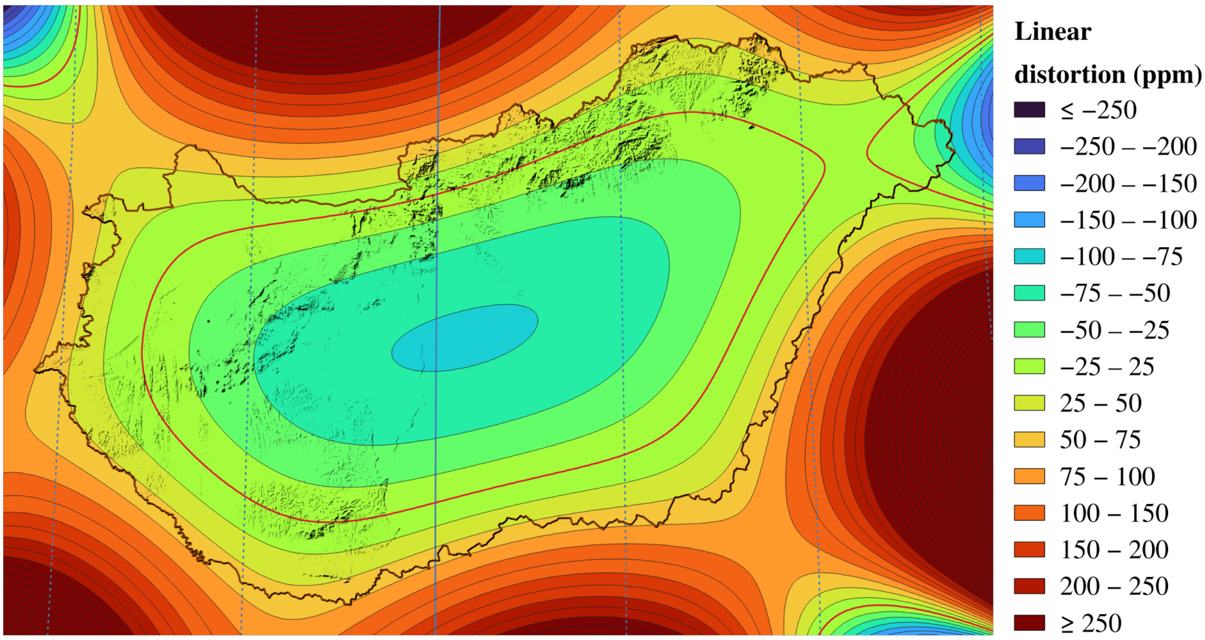

Figure 2. Distribution of linear distortion ($l - 1$) in the resulting mapping. Whole degrees of meridian convergence is shown in blue, standard line of the projection is red.

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