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Are there map projections with coincident standard and secant parallels?

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Abstract:

At the end of April this year, I sent a manuscript to the International Journal of Cartography entitled *A Problem in Basic Cartography*. The editor returned the manuscript to me and asked me to shorten it. I did so (Lapaine 2022), and the adjusted omitted part is the content of this presentation.

In cartographic literature, map projections are usually interpreted by mapping to auxiliary development surfaces, and then these surfaces are developed into a plane. The so-called secant projections, i.e. projections in which the auxiliary surface intersects the Earth's sphere or ellipsoid are especially emphasized. It is stated and taken as a fact without proof that the parallels in which the auxiliary surface intersects the sphere are mapped without distortions. An example of such an approach is the publication Basic Cartography, published several years ago by the International Cartographic Association. This paper, based on a theoretical consideration with an illustration on several examples, concludes that explaining cylindrical projections as mapping on a cylindrical surface is not a good approach, because it leads to misunderstanding important properties of projection. Furthermore, it turns out that the widely accepted facts about secant and standard parallels, which can also be found even in the most recent literature, are wrong and need to be revised. Standard parallels and secant parallels generally do not match.

To accept the argument, we must first establish terminology. Let us suppose that a map projection is a mapping of a sphere or some other curved surface into a plane. Let us suppose that at each point of this mapping we know what the linear scale factor is and that it depends on the direction (Tissot's indicatrix). Let a and b, denote the maximum and minimum values of this factor, respectively (the half-axes of the Tissot's indicatrix).

Let us first note that $a > 0$ and $b > 0$ is always true. Let us see all possible cases when $a = 1$, $b = 1$ or $a = b = 1$ (Lapaine, Menezes 2020):

- If at some point $a = 1$, we can say that this point is locally *equidistant* in the direction of maximum local linear scale factor.
- If at some point $b = 1$, we can say that this point is locally *equidistant* in the direction of minimal local linear scale factor.
- If at some point $a = b = 1$, we can say that this is a point *with zero distortion*, a *zero-distortion point* or a *standard point*.
- If at all points of a line $a = 1$, then it is not generally a line with zero distortion, but we can say that this line is *equidistant* in the direction of maximum local linear scale factor.
- If at all points of a line $b = 1$, then it is not generally a line with zero distortion, but we can say that this line is *equidistant* in the direction of minimal local linear scale factor.
- If at all points of a line $a = b = 1$, then it is a *line with zero distortion*, a *zero-distortion line* or a *standard line*.
- If at all points of an area $a = 1$, then we can say that this area is *equidistant* in the direction of maximum local linear scale factor.
- If at all points of an area $b = 1$, then we can say that this area is *equidistant* in the direction of minimal local linear scale factor.
- The expression $a = b = 1$ cannot be true at all points of a two-dimensional area on the map, as this would mean map projection without distortion. Leonhard Euler first proved that this was not possible in 1777.

So, it is necessary to distinguish between equidistant and standard lines, and therefore equidistant and standard parallels. We also note that in the definitions of equidistant and standard lines no developable surface or intersection is mentioned. We also note that our definition of the standard parallel is consistent with Snyder's (1987) definition:

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'Standard parallels are true to scale and free of angular distortion.' In other words, at each point of the standard parallel $a = b = 1$ holds.

'There are three types of developable surfaces onto which most of the map projections used by the USGS are at least partially geometrically projected. They are the cylinder, the cone, and the plane' (Snyder 1987). What is a developable surface? 'Developable surface is one that can be transformed to a plane without distortion' (Snyder 1987). We accept all of this, but why is the plane mentioned as a developable surface? Nobody knows that.

'When the cylinder is cut along some meridian and unrolled, a cylindrical projection with straight meridians and straight parallels results. The Mercator projection is the best-known example, and its parallels must be mathematically spaced.' (Snyder 1987). But that is not true, Mercator, in the 16th century, did not use a cylindrical surface to define the projection now called Mercator's cylindrical projection. Lambert, in the 18th century, did not use a conical surface to define the projection now called Lambert's conformal conic projection. On the contrary, after deriving the equations of that projection, Lambert mentioned that the map made in that projection could be folded into a cone.

'The cylinder or cone may be secant to or cut the globe at two parallels instead of being tangent to just one. This conceptually provides two standard parallels …' (Snyder 1987). If we accept the definition of standard parallels we have given, and we have seen that Snyder (1987) also accepted it, then we now have an assertion without proof: secant parallels coincide with standard ones. Arguments without evidence are not acceptable in the theory of map projections. It turns out that the claim about the coincidence of secant and standard parallels is in principle wrong.

It is always difficult to introduce changes when long-established custom has created a rut. Namely, it is common to find in the literature on map projections that the basic idea of a secant projection is that the sphere is projected to a cylinder which intersects the sphere at two parallels, say φ_1 north and south. Clearly the scale is now true at these latitudes whereas parallels beneath these latitudes are contracted by the projection and their (parallel) scale factor must be less than one. The result is that deviation of the scale from unity is reduced over a wider range of latitudes. Unfortunately, almost none of the above is true (Lapaine 2022).

Since the various normal aspect cylindrical projections have differently spaced standard parallels, remaining unchanged distances when bending, it is clear that there will generally be no matching of the secant and standard parallels.

Conversely, suppose a cylinder is positioned so that it intersects a sphere in two parallels. Once we develop the surface of the cylinder into a plane, the spacing between the images of the intersecting parallels will not change. If by developing the cylinder surface the secant parallels became standard parallels, then all projections with the same secant parallels would have equally spaced standard parallels. That, of course, is not true.

The paper will give the mathematical conditions that the projection must meet for the standard and secant parallels coincide. These conditions are met only by some cylindrical projections. For example, this is true for the Gall's stereographic projection or some modification of it, such as those to be presented in this paper.

We conclude that explaining cylindrical projections as a projection on a cylindrical surface is not a good approach, as it leads to misunderstanding important projection properties. Standard and secant parallels are often considered identical, but this paper shows that widely accepted facts about these parallels are wrong and need to be revised. This requires a critical approach to the established customs in teaching and researching map projections. In order to prevent misunderstandings in the theory of map projections and their teaching, we recommend avoiding the use of developable surfaces as intermediate surfaces, and thus secant parallels and projections.

Although the need to distinguish between secant and standard parallels has been written about several times (e.g. Lapaine, Menezes 2020, Lapaine, Frančula 2022), it seems that this issue needs to be clarified repeatedly and additionally because many are either unaware of or lack the courage to accept it.

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