

Enclaves and exclaves in tilemaps

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Abstract:

Tilemaps depict geographical regions using geometric enumeration units that are identical in shape and area (e.g. squares or hexagons) and arranged in regular grids. This avoids interpretation errors caused by comparatively small geographical regions that can easily be overlooked or underestimated by the map user.

However, tilemaps pose several cartographically relevant questions regarding production and usability: Both aspects can be understood in terms of graph theory as an assignment problem, in which each real-world reference unit (e.g. each administrative division) must be assigned a corresponding tile. Since the tiles are subject to a predefined regular grid, this assignment can lead to shifts and separations of originally neighbouring reference units, which is why tile maps are often topologically inaccurate. Topological inaccuracy is a particular problem in cases of (semi-) enclaves and (semi-) exclaves (e.g. Vienna within Lower Austria, Berlin within Brandenburg). This article discusses approaches to this problem.

Within the basic principle of the standard method of tilemaps (i.e. both equal size and shape for all reference units) enclaves and exclaves can only be visualized by placing the corresponding tiles next to each other, giving no further creative leeway in the mapping process. However, if we allow one of the two basic principles to be suspended, we can vary the shape or the size of the reference units, which opens a range of design options. In the following, some of these options are presented and empirically evaluated using the example of the German federal states.

The *first step* is to decide which reference units are to be modified in shape or size. On the one hand, this can be done based on (semi-)enclaves and (semi-)exclaves, which corresponds to a topological criterion defined by concepts such as connectivity (e.g. in terms of Betti numbers). Alternatively, non-topological criteria, such as political aspects, can also be used as a basis for modification of tiles, e.g. by adapting only reference units of city-states or capitals. The *second step* is to decide, how the basic geometric enumeration units of a given tilemap should be modified in terms of size and/or shape. In a 2D-scenario, division (e.g. of a square into two equal halves) and insertion (e.g. of a smaller hexagon into a larger one) are the most obvious options (cf. Fig. 1); in a 3D-visualization further alternatives are possible.

Figure 1 shows just a few of these options using the example of the German federal states. Note that there are various options for the standard layout (b), which can be determined mathematically or empirically, cf. Schiewe (2021) and Baumgartner et al. (2021). Also note the additional settings made in Figure 1 (c and d), which cannot be addressed here in further detail this abstract (e.g. the different positions of the inserted square tiles according to the approximate real-world conditions).



Figure 1. States of Germany shown as (a) political map or tilemap with (b) one square tile per state, (c) with tiles of city-states modified in size, or (d) modified in shape and size.

While Figure 1 summarizes some options of how to approach topological inaccuracy in standard tilemaps from a mapmaker's perspective, we also tried to gain a better understanding of how these modifications affect the usability, i.e. how well the different states of Germany can be assigned to the different tiles by map users. Several designs of tilemaps (including those of Fig. 1) were evaluated by 48 probands (bachelor students from the civil engineering and geoinformation department at BHT) within a web-based survey. The following pair of null (H_0) and alternative (H_A) hypotheses was tested:

H₀: User preferences are equally distributed across several squares (or hexagons).

H_A**:** User preferences are not equally distributed across several squares (or hexagons), i.e.: there is exactly one tile that best represents a given federal state.

A chi-squared test was performed to examine H_0 vs. H_A . Results indicate to accept H_A for all map designs tested (statistically significant at the 0.05 level). However, within this wide acceptance of H_A we still can take a closer look on strong and week points of different tilemaps by means of descriptive statistics, analysing the proportion of probands who have decided in favour of a specific allocation. As shown in Figure 2, this proportion varies for the map designs shown initially (Fig. 1) between 50% and 100%, thus indicating both trouble tiles within a given map (e.g. the state of Hessen (HE) in Fig.2; left) and possible advantages of one map over another (e.g. regarding the assignment of the states of Berlin and Brandenburg in Fig. 2; left vs. right).



Figure 2. (In-) consistencies in the assignment of states to tiles modified in size (left), or modified in shape (right) (German acronyms: BW = Baden-Württemberg; BY = Bayern; BE = Berlin; BB = Brandenburg; HB = Bremen; HH = Hamburg; HE = Hessen; MV = Mecklenburg-Vorpommern; NI = Niedersachsen; NW = Nordrhein-Westfalen; RP = Rheinland-Pfalz; SL = Saarland; SN = Sachsen; ST = Sachsen-Anhalt; SH = Schleswig-Holstein; TH = Thüringen)

This paper sheds some light on the handling of (semi-) enclaves and (semi-) exclaves in tilemaps. While we cannot claim to have solved this issue either technically or empirically, the considerations presented are rule-based on principle, which makes them both automatable and scalable on other datasets and thus provide points of reference for future research projects.

References

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