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A review of general perspective projections onto a cylinder

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Abstract:

When discussing perspective map projections onto a cylinder, it usually assumed that axes of revolution of the cylinder and the Earth coincide and that the point of perspective lies on this common axis of revolution. However, Lapaine (1992) showed that other general arrangements are possible, giving a very simple formula for practical calculation. On the other hand, Lapaine has not discussed the practical usability of such mappings. This presentation tries to explore it.

It is very important to highlight that these general perspective projections onto a surface of the cylinder are *not* cylindrical map projections. They do not fulfil the definition of cylindrical map projections provided by Lee (1944). (I.e., they do not map parallels to parallel straight lines in the normal aspect.)

If we define the normal aspect of a map projection as the aspect having the simplest mathematical description (Lee, 1944, Wray, 1974), which correspond to the aspect having the greatest degree of symmetry, then the normal aspect of the general perspective map projection onto a cylinder is the aspect, in which the axes of revolution of the cylinder and the sphere are parallel to each other.

We will assume that the centre of projection (the "light source") is at the origin. Axis z is rotated so that it is parallel to the axis of revolution of the cylinder. Axis x is perpendicular to z and intersects the axis of revolution of the cylinder. Axis y is placed to have axes xyz form a right-handed Cartesian coordinate system. The centre of the sphere is at (x_0, y_0, z_0), its metapoles are located parallel to axis z. The radius of the cylinder is r, its axis of revolution intersects axis x at abscissa rc (this results in that parameter r merely scales the map projection). Thus, we can uniquely describe a general cylindrical map projection (disregarding the scale) by only four scalar parameters. Using this parametrization and assuming that the Earth is a unit sphere, the formulae of Lapaine (1992) will simplify to:

$$x = x_0 + \cos\varphi' \cos\lambda' \tag{1}$$

$$y = y_0 + \cos\varphi' \sin\lambda' \tag{2}$$

$$z = z_0 + \sin\varphi' \tag{3}$$

$$t = \frac{cx \pm \sqrt{c^2x^2 - \left(x^2 + y^2\right)\left(c^2 - 1\right)}}{x^2 + y^2} \tag{4}$$

$$X = r \arctan \frac{ty}{tx - c} \tag{5}$$

$$Y = rtz \tag{6}$$

Where X and Y are the map coordinates, arctan is the function atan2 of conventional programming languages, φ' and λ' are metalatitude and metalongitude, respectively.

We should note that t above must be positive, as it is related to the position of the mapped point along the projection ray. If the expression under the square root is negative, then t is complex, meaning that the projection ray has no intersection with the cylinder. This can happen only if the point of perspective is outside the cylinder ($|c| > 1$). If the sphere intersects at least one of the two planes containing axis z and tangent to the cylinder, then parts of the sphere outside these planes cannot be mapped. Otherwise, t has two solutions. If $t < 0$, then the centre of projection (origin) falls between the point and its image, so the image gets mirrored. This should be avoided (a mirrored map is not useful for orientation) and the same image without mirroring can be produced by reversing the sign of c. If there are no positive solutions for t, the point cannot be mapped. This is possible only if the point of perspective is outside the cylinder ($|c| > 1$). If there are two positive solutions for t, we can make an arbitrary choice, as the projection ray intersects the cylinder twice. Furthermore, if the centre of projection is outside the sphere ($x_0^2 + y_0^2 + z_0^2 > 1$), then some projecting rays intersect the sphere twice, meaning that two different areas are mapped onto the same area on the cylinder. In this case, we must decide whether we want to map the near or the far side of the Earth.

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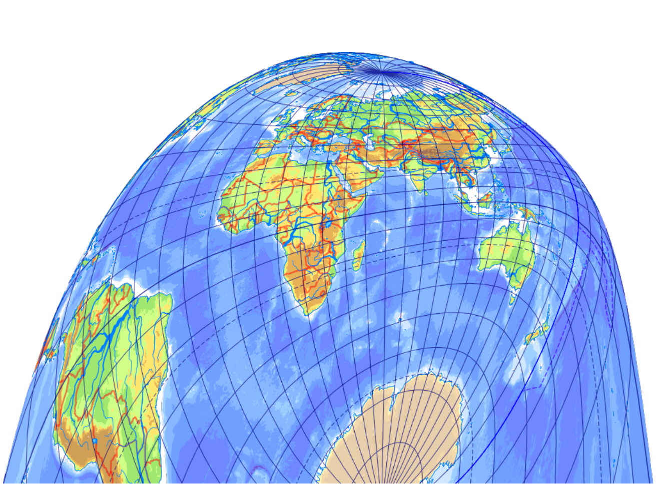
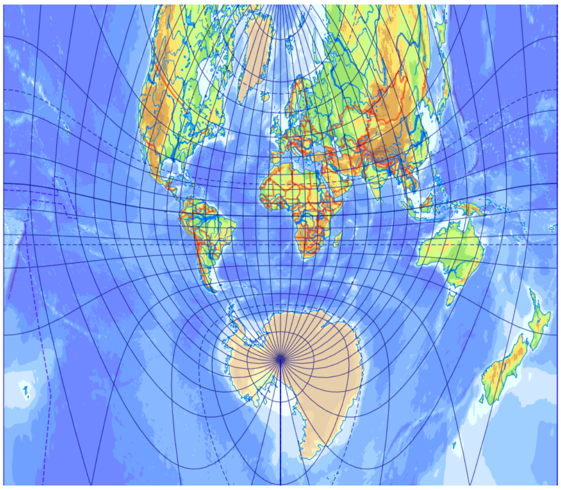
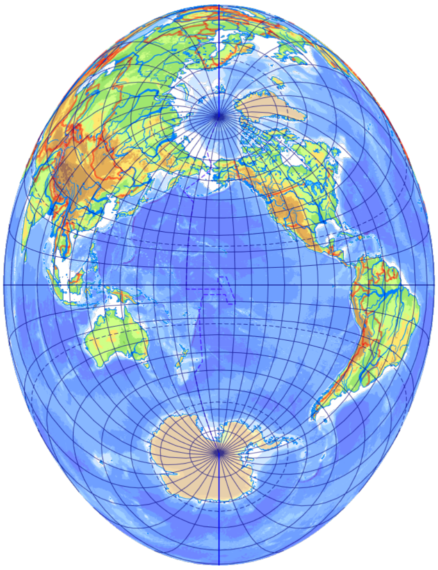
A few examples of such general perspective maps onto a cylinder are examined for usability in practical cartography in Fig. 1. The first image shows that if $y_0 \neq 0$, then the map will not be symmetrical about the vertical axis, which is very unusual, so it should not be used. The second image shows that if the centre of projection is inside the sphere, the map will be very distorted, usable only along equatorial regions. On the third image, the reader can see that far-side perspectives are rather suitable for nearly circular regions, resulting in an image very similar to planar perspectives. Similar maps were analysed by Alashaikh et al. (2014), showing that they are very useful for oval regions. The near-side perspective is probably the most interesting application, as it can show the sphericity of the Earth while reducing distortions along the metaequator compared to the orthographic projection. This is possibly the most useful application of general perspectives onto a cylinder. The fourth image is such a near-side perspective projection in oblique transverse aspect (the metaequator runs along meridian 15°) optimized to show Africa.



(a) $x_0 = 0.5; y_0 = 0.8; z_0 = -0.7; c = 0.5$



(b) $x_0 = 0.4; y_0 = 0; z_0 = 0.6; c = 0.3$



(c) $x_0 = 1.25; y_0 = 0; z_0 = 0; c = -0.48$

(d) $x_0 = -1.75; y_0 = 0; z_0 = 0; c = -1.72$ (transverse)

Figure 1. Some examples of general perspective projections onto a cylinder.

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